Analysis of Safe and Failure Mode Regimes of Dielectric Elastomer Actuators

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***Abstract*—Dielectric elastomer actuators (DEAs) are promising for bionic and robotic applications. Reliability requirements in such applications necessitate better knowledge of the safe operation regimes and possible failure modes of DEAs. Elastomers consist of a network of entangled and cross-linked polymer chains, able to sustain spatially varying stress and strain fields, accompanied by stored elastic energy. Here we introduce a rigorous thermodynamic description of DEAs, based on a statistical mechanical model to gain insight for material development and optimization. We investigate multistability effects in the phase diagrams of elastomer actuators and combine our model with capacitive extensometry experiments.**

I. Introduction

Dielectric elastomer actuators (DEAs) are based on deformable capacitors with highly compliant electrodes. When a voltage is applied between these electrodes, a significant Maxwell stress is acting on the DEA. Thereby the elastomer film is squeezed in the thickness direction and expanded in the film plane, due to the incompressibility of the material. A failure mode of DEAs which is currently subject of theoretical [1, 2] and experimental investigations [3, 4] is the pull-in instability, which frequently occurs in electrostatically driven systems. Early experimental observations of such mechanical instabilities are given in [5]. Since DEAs also work on electrostatic principles, they are likely showing pull-in instabilities as well. In fact recent theoretical and experimental work indicates such failure modes in DEAs [1-4]. Combining theoretical modeling with experimental investigations should improve understanding of stable and failure mode regimes of DEAs.

In this paper, a strict thermodynamic treatment of electromechanical actuators is used to model suchlike systems and analyze them with respect to mechanical instabilities. Microelectromechanical devices (MEMs) are well known to show an inherent electromechanical instability [6] and therefore provide a suitable introduction to the topic. Fig. 1 (a) schematically shows a simple actuator, consisting of two capacitor plates where one of the plates is fixed and the second one is connected to a spring. When applying a charge Q, the capacitor plates will be attracted due to

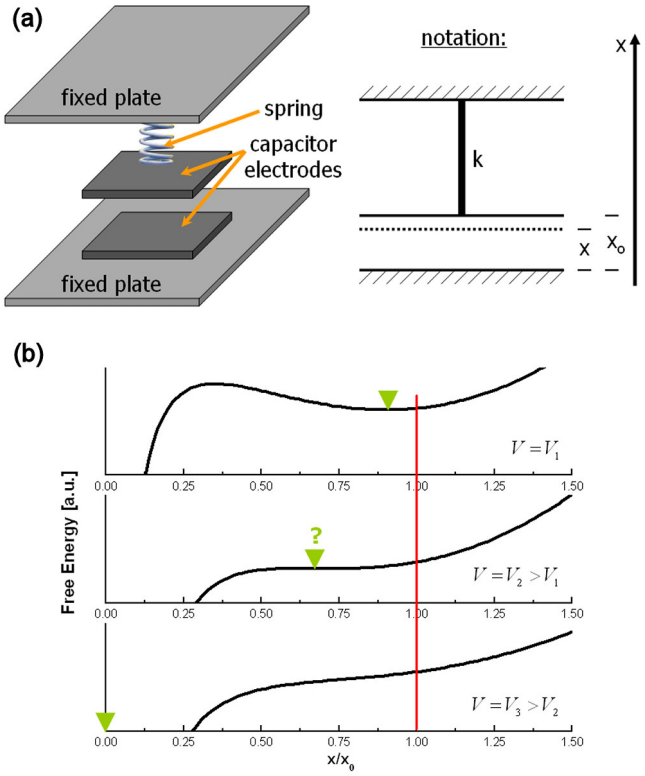

Figure 1. (a) Shematic illustration of a simple micromechanical actuator. (b) Free energy versus spring extension ratio of a suchlike actuator in the voltage controlled configuration. Transition from a stable minimum at $V=V_1$ to a critical point at $V=V_2$, followed by a collaps of the capacitor plates at a voltage exceeding V_2.

electrostatic forces, which results in an elongation of the spring. The total amount of energy stored in such a system is given by

$$E(x,Q)=\frac{1}{2}k(x_0-x)^2+\frac{1}{2}\frac{Q^2}{C} \qquad (1)$$

with the capacitance C and the spring constant k. Here, x_0 denotes the initial separation of the capacitor plates when no

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charge is applied. Equation (1) correctly accounts for the energy of the system in a Q-controlled case. When connecting the capacitor plates to a voltage source, the applied voltage V will determine the deformation of the actuator. In this configuration, the free energy of the system is obtained from (1) by a Legendre-transformation from the extensive variable Q to its conjugated intensive quantity V:

$$F(x,V) = E(x,Q) - QV \tag{2}$$

For a plate capacitor, together with $Q = CV$, this results in

$$F(x,V) = \frac{1}{2}k(x_0 - x)^2 - \frac{\varepsilon_0 A}{2x}V^2 \tag{3}$$

with the vacuum permittivity ε_0 and the area of the capacitor plates A. The equilibrium deformation positions of this actuator are obtained from the extrema of the free energy function, whereat a minimum corresponds to a stable actuator state and a maximum to an unstable one. This implies an important advantage of the thermodynamic approach: The possibility to easily perform a stability analysis of an actuator. Critical stability is achieved, when a saddle point in the free energy function is reached. Fig. 1 (b) illustrates the free energy function for three applied voltages $V_1 < V_2 < V_3$, versus spring extension ratio x/x_0. Fig. 1 (b) clearly shows a stable minimum in the free energy at an applied voltage of $V = V_1$, reaching a critical point when the voltage is increased up to $V = V_2$. Any perturbation of the system at this point causes the capacitor plates to collapse, leading to an electrical shortcut as final result. No stable minimum in the free energy function can be found for voltages exceeding this critical value. Critical stability occurs, when both the first and second derivative of the free energy function with respect to x vanish:

$$\left(\frac{\partial F(x,V)}{\partial x}\right)_V = 0 \quad \text{and} \quad \left(\frac{\partial^2 F(x,V)}{\partial x^2}\right)_V = 0\,. \tag{4}$$

Solving this pair of equations results in a critical value $x = (2/3)x_0$, leading to a mechanical instability of the actuator as soon as the applied voltage is large enough to expand the spring up to the afore mentioned value.

II. Modeling Circular DEAs

Dielectric elastomer actuators show impressive area expansions when actuated with large electric fields, what makes them promising for bionic and robotic applications [7]. Area expansion rates of up to 300% have been reported in [8]. This necessitates the use of hyperelasticity models to describe the large elastomer deformations.

The Arruda-Boyce eight chain model [9] is based on a statistical mechanical treatment of the highly flexible molecular network of elastomers. It implies that all chains deform in the same way. This simplifies averaging and results in an inverse Langevin-type free energy function, accounting for the limited extensibility of the chain molecules. The free energy density of an elastomer is then

$$F = Nk_BTn\left(\tilde{r}\,\mathscr{L}^{-1}(\tilde{r}) + \ln\left(\frac{\mathscr{L}^{-1}(\tilde{r})}{\sinh\left(\mathscr{L}^{-1}(\tilde{r})\right)}\right)\right) \tag{5}$$

when using the abbreviation $\tilde{r} = (3n)^{-1/2}\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}$. N is the number of molecular chains per unit volume, k_B and T denote the Boltzmann constant and the absolute temperature. The finite extensibility of a molecular chain is determined by the number of chain links n.

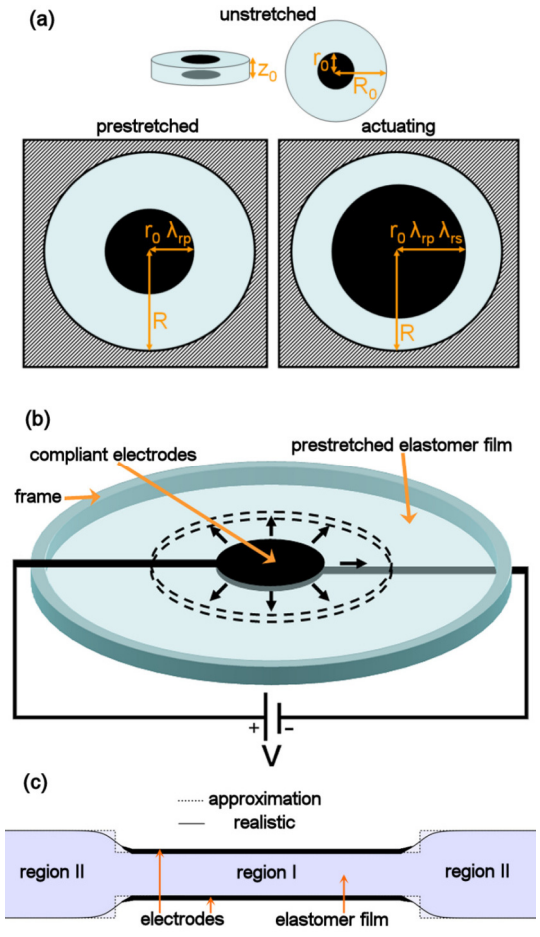

Figure 2. (a) Geometry of a circular DEA. An elastomer film of radius R_0 and initial thickness z_0 is prestretched to a rigid frame of radius R, defining the prestretch λ_{rp}. (b) Actuation scheme of circular DEAs. (c) Cross section of the elastomer film. When actuated, the elastomer film sandwiched between the compliant electrodes (region I) is compressed, whereas the surrounding elastomer (region II) expands in its thickness direction. The border between this two regions is modelled by a stepfunction, beeing a reasonable approximation for not too small electrode areas.

Equation (5) uses the inverse of the Langevin function defined as $\mathscr{L}(x) = \coth(x) - 1/x$. The affine network deformation assumption allows for implementing the principal stretch ratios λ_i directly into the molecular theory. Furthermore, we now may apply this model to calculate the free energy density of a circular DEA.

The geometry of a suchlike designed actuator is depicted in Fig. 2. An elastomer film of radius R_0 and initial thickness z_0 is symmetrically prestretched to a rigid frame of radius R by a factor of $\lambda_{rp} = R/R_0$. The central part of the elastomer film is sandwiched between two highly compliant, circular electrodes, forming a plate capacitor with strain dependent area and thickness. When a voltage is applied to the electrodes, a significant Maxwell stress is acting on the DEA. Thereby, the elastomer film is squeezed in its thickness direction and expands in the film plane due to the incompressibility of the material. The total radial stretch ratio is now given by $\lambda_r = \lambda_{rp}\lambda_{rs}$, where λ_{rs} denotes the secondary radial stretch ratio due to the present Maxwell stress. Fig. 2. (b) schematically illustrates the actuation process of a circular DEA. A cross section of the elastomer film is shown in Fig. 2 (c), depicting the transition in the elastomer film from the area sandwiched between the compliant electrodes (region I) to the surrounding elastomer (region II). For modeling the DEA, this transition will be treated as a step in the elastomer thickness, being a reasonable approximation for not too small electrode diameters. From the geometry, together with the incompressibility condition $\lambda_r^2\lambda_z = 1$, we now find for the chain length in elastomer region I

$$\tilde{r}_{\mathrm{I}} = \frac{1}{\sqrt{3n}}\sqrt{2\lambda_{rp}^2\lambda_{rs}^2 + \lambda_{rp}^{-4}\lambda_{rs}^{-4}}\,. \qquad (6)$$

Assuming the aforementioned step in the elastomer thickness at the transition from region I to region II, we can obtain the principal strain variables for the elastomer in region II. Considering volume conservation, together with the purely geometrical ratio of frame- to electrode radius $\kappa = R_0/r_0$, one derives

$$\tilde{r}_{\mathrm{II}} = \frac{1}{\sqrt{3n}}\sqrt{\frac{2\left(\kappa^2 - \lambda_{rs}^2\right)\lambda_{rp}^2}{\left(\kappa^2 - 1\right)} + \frac{\left(\kappa^2 - 1\right)^2}{\left(\kappa^2 - \lambda_{rs}^2\right)^2\lambda_{rp}^4}}\,. \qquad (7)$$

The free energy density F_{I} and F_{II} of the elastomer in both regions I and II is obtained by substitution of (6) and (7) into (5).

In the voltage controlled configuration, the overall free energy density of a circular DEA is given by the sum of the free energy density of both elastomer regions as well as the electrostatic energy density of the deformable capacitor in its V-controlled representation. We then find with $\lambda_r = \lambda_{rp}\lambda_{rs}$:

$$F(\lambda_r, V) = F_{\mathrm{I}}(\lambda_r) + \left(\kappa^2 - 1\right)F_{\mathrm{II}}(\lambda_r) - \frac{\varepsilon_0\varepsilon}{2z_0^2}V^2\lambda_r^4\,. \qquad (8)$$

Here, ε is the permittivity of the elastomer and $\kappa^2 - 1$ accounts for the volume fraction $V_{\mathrm{II}}/V_{\mathrm{I}}$.

The free energy density of a circular DEA given in (8) is used to analyze the stability of this electromechanical actuator. The form of the phase diagram is mainly determined by the two geometrical parameters λ_{rp} and κ, as well as by the material specific number of chain links n, whereas the number of molecular chains per unit volume N primarily decides on the voltage necessary for a specific deformation state.

Fig. 3 (a) depicts the free energy density for an actuator with $\lambda_{\mathrm{rp}} = 1.9$, $\kappa = 9.5$ and a typical number of 116 chain links n. For $V < V_{\mathrm{crit}}$, two stable minima at different radial stretch ratios occur in the phase diagram. When increasing the voltage, the DEA's electrodes expand until a critical stretch ratio at an applied voltage of $V = V_{\mathrm{crit}}$ is reached. Here, the left minimum of the free energy density turns unstable, causing an expansion process of the DEA until the second stable minimum is reached at a significantly higher stretch ratio. The third curve shows the free energy density

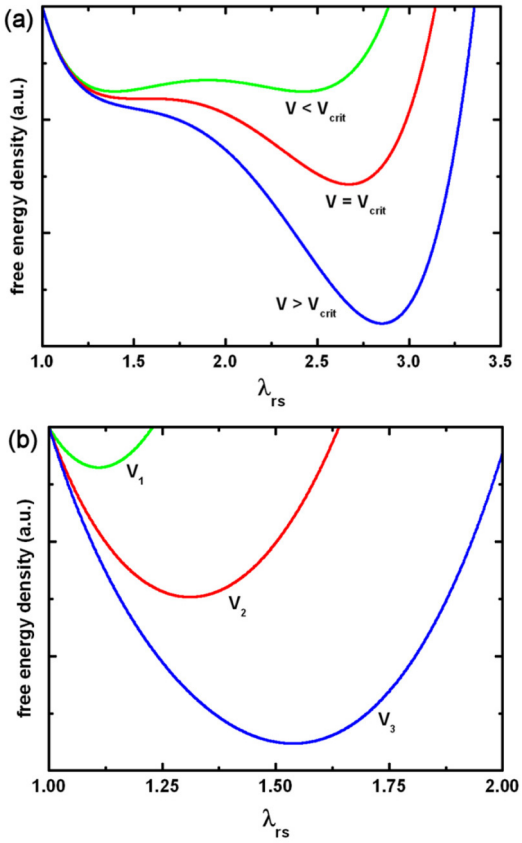

Figure 3. (a) Free energy density of a circular DEA versus radial stretch ratio λ_{rs} for a radial prestrech of 1.9. The phase diagram shows two minima up to a critical voltage. (b) Prestretching the elastomer by a factor of 2.1 prevents the phase diagram from beeing bistable, independent of the applied voltage.

function for $V > V_{crit}$ with only one stable minimum.

When increasing the prestretch of the elastomer film up to 2.1, while leaving the other parameter values unchanged, the phase diagram changes its form significantly, as shown in Fig. 3 (b). Now, independent of the applied voltage, only a single stable minimum occurs in the free energy density.

A condition equivalent to (4) can be used to show numerically, that for a given number of chain links, prestretching the elastomer film up to a certain value prevents the appearance of a critical point in the phase diagram.

III. Experimental Results

Since DEAs are deformable capacitors, it is reasonable to characterize them electrically. An experimental technique to investigate the transient strain in circular DEAs is presented in [10]. It allows for keeping *in situ* track of the rapidly changing strain values in the vicinity of a critical point in the phase diagram and thus helps to understand the physical mechanisms underlying the pull-in instability.

A DEA with a radial prestretch of 1.9 and a frame- to electrode radius ratio of 9.5 was prepared on the basis of the 3M VHB 4905 elastomer tape with an initial thickness of 500 µm. The electrodes were prepared using a mixture of carbon black and silicon oil, which forms highly compliant electrodes [11].

According to the presented theoretical model, this geometry is likely to show a pull-in instability for a variety of elastomer materials. Fig. 4 shows the transient radial strain of the DEA when voltages of 2.5 kV and 3.5 kV are applied to drive the DEA in the instability region. The dynamics of the strain values reveals an inflection point at a constant voltage of 3.5 kV, followed by an abrupt, rapid increase in the area expansion until dielectric breakdown occurs as the final failure mode. The photos in Fig. 4 illustrate the formation of a wrinkled elastomer film region, which is a further indication of rapid area increase. The measured behavior can therefore be interpreted as an experimental monitoring of the pull-in instability. However, a stabilization of the expansion process at a high radial stretch ratio could not be observed, because the electric field exceeded the dielectric strength of the thin wrinkled region in the VHB 4905 elastomer film.

IV. Conclusion

We have developed a theoretical model for circular DEAs, based on a rigorous thermodynamic treatment and a hyperelasticity model, comprising the underlying molecular network of the elastomer to analyze safe and failure mode regimes of DEA operation. An experimental observation of the pull-in instability affirms the predictions of this theory.

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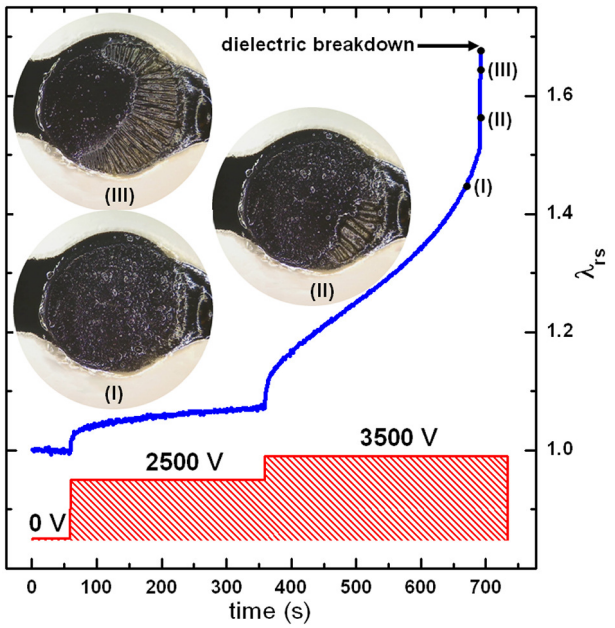

Figure 4. Transient radial strain data of a circular DEA, revealing the slow viscoelastic development of the pull-in instability. At an applied voltage of 3.5 kV, the radial stretch ratio first tends to stabilize, after reaching the inflection point the electrode area increases rapidly until dielectric breakdown occurs as a final failure mode. The inset photos depict the formation and evolution of a wrinkled film area in the actuator.

References

[1] X. Zhao, W. Hong, and Z. Suo, "Electromechanical hysteresis and coexistent states in dielectric elastomers", Phys. Rev. B, vol. 76, nr. 13, pp. 134113-1−134113-9, 2007

[2] X. Zaho and Z. Suo, "Method to analyze electromechanical stability of dielectric elastomers", Appl. Phys. Let. vol. 91, pp. 061921-1 - 061921-3, 2007

[3] J. Plante and S. Dubowsky, "Large-scale failure modes of dielectric elastomer actuators", Int. J. Solid Str., vol. 43, pp. 7727-7751, 2006

[4] S. Bauer and M. Paajanen, "Electromechanical characterization and measurement protocol for dielectric elastomer actuators", Proc. SPIE, vol. 6168, pp. 698-706, 2006

[5] K. H. Stark and C. G. Garton, "Electric strength of irradiated polythene", Nature, vol. 176 nr. 4495, pp. 1225-1226, 1955

[6] H. C. Nathanson, W. E. Newell, R. A. Wickstrom and J. Ransford Davis, "The resonant gate transistor", Jr., IEEE Trans. Electron Devices, vol. 14, pp. 117-133, 1967

[7] G. Kofod, M. Paajanen and S. Bauer, "Self-organized minimum-energy structures for dielectric elastomer actuators", Appl. Phys. A, vol. 85, pp. 141-143, 2006

[8] R. Pelrine, R. Kornbluh, Q. Pei and J. Joseph, "High-speed electrically actuated elastomers with strain greater than 100%", Science, vol. 287, pp. 836-839, 2000

[9] E. M. Arruda and M. C. Boyce, "A three-dimensional constitutive model for the large stretch behavior of rubber elastic materials", J. Mech. Phys. Solids, vol. 41 nr. 2, pp. 389-412, 1993

[10] C. Keplinger, M. Kaltenbrunner, N. Arnold and S. Bauer, "Capacitive extensometry for transient strain analysis of dielectric elastomer actuators", Appl. Phys. Let., vol 92, pp. 192903-1 – 192903-3, 2008

[11] G. Kofod, "Dieletric elastomer actuators", PhD thesis, Riso-R-1286, Denmark , 2001